TUNED TRANSFORMERS . . .

Design of these electronic units simplified by means of universal performance curves

PART I

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IN THE age of electronics we visualize a myriad of vacuum tubes accomplishing feats of almost intelligent character. We feel the need of knowing how these tubes function and this immediately presents the question—into what sort of electric circuits will these tubes work?

One of the functions which electronic devices will perform will be the selection of a proper signal for action. In radio engineering, selection immediately suggests frequency selectivity and this method of discrimination will undoubtedly find application in many other electronic devices. Not only will this discrimination be applied to electric currents and voltages but also to mechanical and sound vibrations. By the proper analogs our information on electric cricuits may be used for mechanical design.[1]

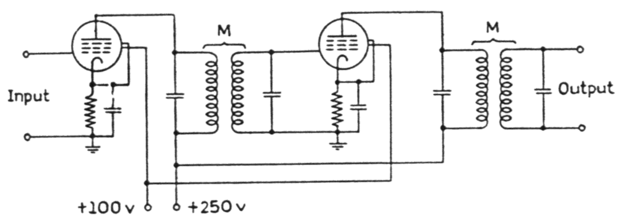


Fig. 1. An amplifier stage containing two tuned transformers

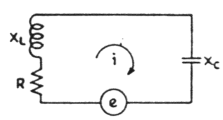


Fig. 2. A single tuned circuit

Selectivity

Tuned circuits have long been used in radio equipment to provide this frequency selection. The most elementary selective device is a single resonant circuit. For a greater degree of selectivity cascaded resonant circuits are used. If the cascaded circuits are prevented from reacting in a backward direction by insertion of amplifier tubes between circuits, they retain their simple selective character. However, if two or more circuits are connected together so that a later one can influence an earlier one through a common coupling, the selection against frequency is no longer a simple resonance curve, but may pass a range of frequencies and attenuate or reduce the response to other frequencies. The shape of the frequency-selective characteristic then depends on the coupling between circuits and we have what may be called a tuned transformer.

Tuned transformers, consisting of two resonant circuits with common coupling, provide, with appropriate design, nearly all the frequency-selective characteristics of a band-pass nature which may be desired. Sharper selection against undesired frequencies than is obtainable with one such transformer is obtained by cascading transformers, using amplifier tubes to isolate later transformers from earlier ones. An amplifier stage containing such transformers is illustrated in **Fig. 1**.

Design Curves

A set of design curves can be developed for the selective performance of those transformers.[2] The development of the formula for the curves will be briefly reviewed.

A single tuned circuit, **Fig. 2**, has a series impedance $Z=e/i=R+j\,(X_L-X_c)$. Calling resonant frequency f_0 where $X_0=X_L=X_c$, then for any frequency f

$$Z=R\,(1+jt),\ \text{if}\ t=Q_0\,(f/f_0-f_0/f),\ Q_0=X_0/R \qquad (1)$$

where t is the tangent of the phase angle between e and i, and Q_0 is the "quality factor" of the coil at f_0. Quality factor or Q is widely used in coil measurement and is the ratio of reactance to resistance at a specified frequency.

This single circuit has a parallel impedance across X_c of

$$Z=\frac{X_c\,(R+jX_L)}{R+j\,(X_L-X_c)}=jX_0\left(\frac{f_0/f+jQ_0}{1+jt}\right) \qquad (2)$$

Assuming Q_0 is five or more the magnitude of this impedance will be very nearly

$$Z=\frac{X_0\,Q_0}{\sqrt{1+t^2}} \qquad (3)$$

The variable t may be further simplified and interpreted. By ordinary algebraic manipulation

$$t=Q_0\,(\Delta f/f_0)\,(1+f_0/f),\ \text{if}\ \Delta f=f-f_0 \qquad (4)$$

(1)"Applications and Limitations of Mechanical-electrical Analogies, New and Old," by John Miles, *Journal of the Acoustical Society of America*, vol. 14, p. 183, January, 1943.

(2)"Universal Performance Curves for Tuned Transformers," by J. E. Maynard, *Electronics*, vol. 10, p. 15, February, 1937.

Making the approximation $f_0/f = 1.0$ (near f_0)

$$t = 2\Delta f\, Q_0/f_0 \qquad (5)$$

The impedance of the single circuit may, therefore, be calculated in terms of frequency deviation from resonance for a coil of given Q. If the circuit contains a load resistance (R_L) in parallel with the capacitor, this may be taken into consideration in writing the original parallel impedance, in which case we will find that the result can be written

$$Z' = \frac{X_0 Q_0'}{\sqrt{1+(t')^2}}, \text{ where } \frac{Q_0'}{Q_0} = \frac{R_L}{Z_0 + R_L}, \; t' = \frac{2\Delta f Q_0'}{f_0}, Z_0 = X_0 Q_0 \quad (6)$$

so that the net effect is a change in Q_0. An amplifier tube working into such an impedance may be analyzed by the conventional generator method using a voltage μe_g driving the impedance Z through a plate resistance R_p. If this is done, the voltage across the impedance Z will be

$$e_Z = (\mu/R_p)\, e_g \left(\frac{R_p Z}{R_p + Z}\right) = g_m\, e_g\, Z' \qquad (7)$$

where g_m is the tube's mutual conductance and Z' is the impedance of the circuit determined by a Q_0' which includes R_p in parallel with the circuit. The ratio e_Z/e_g is the voltage gain of the amplifier.

If we take the ratio of voltage gain at resonant frequency f_0 to the voltage gain at any frequency f, this will be a measure of the selectivity of the circuit

$$U = G_0/G_f = g_m Z_0/g_m Z = \sqrt{1+t^2} \qquad (8)$$

This "attenuation" U is the "number of times down" or the reduction ratio between the outputs from a resonant signal and some other equal signal at a frequency deviation of Δf from resonance.

In order to cover a large ratio of attenuation, selectivity curves are usually plotted using a vertical logarithmic scale for attenuation and a horizontal linear scale for frequency deviation, as shown in **Fig. 3**. Certain advantages (which will be apparent later) are obtained from a plot using logarithmic scales on both axes, as shown in **Fig. 4**. For design purposes our approximations result in a symmetrical curve about resonance so the half curve which results from the use of a log-log chart is sufficient.

Any coupling arrangement between two circuits may be classified as a Y or Δ arrangement of circuit elements and, in turn, either may be reduced to an equivalent Y circuit. A general circuit, as shown in **Fig. 5**, may therefore be used for analysis. This circuit may represent a transformer, such as shown in **Fig. 1**, in which Z_1 is the series impedance of the primary circuit and Z_2 the series impedance of the secondary circuit, while Z_m is the mutual inductive reactance equivalent to the effective inductive coupling between coils. The output voltage will be the reactive drop produced by secondary current in the secondary capacitor.

The circuit of **Fig. 5** may be solved for the ratio between input voltage e and output current i_2.

$$e/i_2 = \frac{Z_1 Z_2 - Z_m^2}{Z_m} \qquad (9)$$

Applying this to the particular case of inductive coupling and output voltage e_2 across a secondary capacitive reactance X_{c2} and using the series impedance $Z = R\,(1+jt)$, we arrive at the voltage ratio

$$e/e_2 = \frac{R_1 R_2}{X_{c2} X_m}\left[(1+jt_1)\,(1+jt_2) + \frac{X_m^2}{R_1 R_1}\right] \qquad (10)$$

Using the definition coefficient of coupling

$$K = X_m/\sqrt{X_1 X_2}$$

the reactances involved in this case being those of the inductances, the term $X_m^2/R_1 R_2$ becomes $K^2 Q_1 Q_2$. If

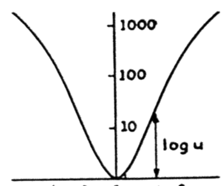


Fig. 3. Selectivity curve plotted with a logarithmic vertical scale (attenuation) and a linear horizontal scale (frequency deviation)

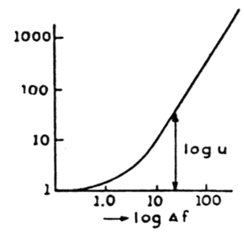


Fig. 4. Selectivity curve plotted using logarithmic scales on both axes. Only half of the curve is shown since it is symmetrical about resonance

we assume for the moment that primary and secondary are identical and determine the magnitude of this voltage ratio without regard to phase, then

$$|e/e_2| = \frac{R^2}{X_m X_c}\sqrt{(1+K^2Q^2-t^2)^2+4t^2} \qquad (11)$$

At resonance ($t=0$) the voltage $|e_2|$ will be

$$|e_2| = \frac{X_m X_c e}{R^2}\left(\frac{1}{1+K^2 Q^2}\right) \qquad (12)$$

Now the attenuation of this circuit to constant excitation voltage e for off-resonant frequencies will be obtained by dividing resonant voltage e_2 by off-resonant voltage e_2 (obtained from Equation (11)), or

$$U = \frac{\sqrt{(1+K^2Q^2-t^2)^2+4t^2}}{1+K^2Q^2} \qquad (13)$$

if we make the approximation that X_m and X_c are essentially constant near f_0.

Fig. 6 represents a circuit being driven by a constant current i_c which may be constant, for our purposes, with respect to frequency. The voltage drop $i_c Z_a$ is equal to $iZ_a + iZ_b$ and so may be considered equivalent to the driving voltage e we have used, if we neglect the variation of Z_a with frequency. Z_a has been, in our analysis, the capacitive reactance in the primary circuit. Z_b is the remainder of the circuit and may vary in any manner with frequency. Referring to

Equation (7), the terms $g_m e_g$ are by definition a-c plate current in the amplifier tube and this corresponds to the current i_c of **Fig. 6** if the plate resistance is absorbed in Z_b. In other words, the grid voltage of the amplifier driving the transformer may now be linked with the secondary voltage e_2 by replacing e by $-jg_m e_g X_c$.

The simplicity of the results obtainable for identical primary and secondary leads to the desire for an equally simple result for different primary and secondary. By making the appropriate conversions this may be achieved.

The geometric mean Q is

$$Q_G = \sqrt{Q_1 Q_2} \tag{14}$$

and the arithmetic mean Q

$$Q_A = (Q_1 + Q_2)/2 \tag{15}$$

Now, if we define a new Q which will give the same attenuation curve given by Q_1 and Q_2 when that Q is used in both primary and secondary, that new Q will be

$$Q = Q_G\,(Q_G/Q_A) \tag{16}$$

This may be shown by making the substitutions which

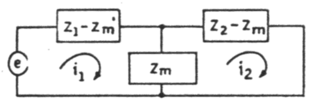


Fig. 5. This general circuit arrangement can be used to represent any coupling arrangement between two circuits

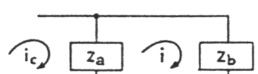


Fig. 6. Circuit driven by a constant current (with respect to frequency)

recast our equations in terms of this new Q as follows:

$$S = (Q/Q_1)\,t_1 = (Q/Q_2)\,t_2 \tag{17}$$

$$1 + C^2 = (Q_G/Q_A)^2 + K^2Q^2 \tag{18}$$

These substitutions applied to Equation (10) give us

$$e/e_2 = (Q_A/Q_G)^2 \left(\frac{R_1 R_2}{X_{c2} X_m}\right)(1 + C^2 - S^2 + 2JS) \tag{19}$$

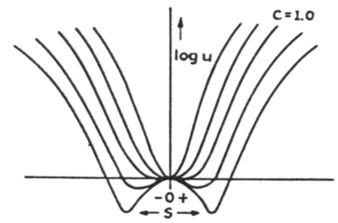
Taking magnitudes without regard to phase and reducing this to a formula for attenuation as in Equation (13) we have

$$U = \frac{\sqrt{(1 + C^2 - S^2)^2 + 4S^2}}{1 + C^2} \tag{20}$$

which is a universal expression for attenuation which may be applied to any pair of coupled resonant circuits. The similarity between Equations (13) and (20) allow us to interpret the symbols in Equation (20) which holds for circuits of unequal Q. The symbol S is a function of frequency, just as t is, and may be readily shown to be similar to the definition of t in Equation (5)

$$S = 2\Delta f Q/f_0 \tag{21}$$

S may be thought of as a frequency-selective variable. The symbol C by comparison with (10) and (18) is seen to be a function of coefficient of coupling K. By applying the usual differential test for maxima and minima to Equation (20), using the frequency variable S as independent and the attenuation U as the dependent variable, it will be found that a single attenuation minimum is obtained for all values of the coupling parameter C of 1.0 or less, and that for values of C of more than 1.0 a maximum and two minima are obtained. The coupling corresponding with $C = 1.0$ is called critical or optimum coupling with respect to the shape of the selectivity curve. If Equation (20) were plotted as in **Fig. 3** a family of curves as shown in **Fig. 7** would be obtained.



($C = 1.0$ refers to the center curve.)

Fig. 7. A family of selectivity curves plotted from Equation (20)

For design purposes it is more convenient to plot these curves on log-log paper in which case approximately half of each curve is seen. The second part of this article will deal with some characteristics of the curves as drawn in a design chart and with a practical problem in which they are used.

(To be continued)

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PART II

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THE selectivity curves plotted in the chart, **Fig. 8**, show several characteristics which, when evaluated, are an aid in rapidly estimating the selectivity obtainable from a given number of circuits. This information will, in addition, assist in obtaining interpolated curves on the chart.

The first characteristic which is immediately apparent in the chart is the asymptotic nature of the curves. All the curves can be roughly represented by two straight lines, one being the unit attenuation line and the other being the slope asymptote, **Fig. 9**. The equation of the asymptote may be obtained by investigating the value of attenuation U as the selective variable S becomes very large. For the single circuit $U=\sqrt{1+S^2}$ the equation of the asymptote will be

$$U=S \tag{22}$$

On log-log paper, this is represented by a 45-deg. line starting from $U=1$, $S=1$. If two single circuits are used in cascade (no back coupling), this attenuation ratio will be applied twice to the signals so that the resultant attenuation will be $U=1+S^2$ and the asymptote $U=S^2$. The asymptote on log-log paper will be plotted as $\log U=2\log S$ and therefore will have a slope of 2/1. It is apparent that if this reasoning is carried through for N cascaded single circuits, the asymptote as plotted in logarithmic units will be

$$\log U=N\log S \tag{23}$$

so that the slope of the asymptote will be N. The intercept with unity attenuation will still occur at $U=1$, $S=1$.

For a coupled pair of circuits, we have found the attenuation to be $U=\dfrac{\sqrt{(1+C^2-S^2)^2+4S^2}}{1+C^2}$. To simplify the notation we will let $1+C^2=S_0^2$. The attenuation formula is then

$$U=\frac{\sqrt{(S_0^2-S^2)^2+4S^2}}{S_0^2} \tag{24}$$

Now, if S becomes very large, we reach the asymptote

$$\log U=2\log S-2\log S_0$$

This is represented by a straight line on log-log paper which starts ($U=1$) from $S=S_0$ and has a 2/1 slope. This is gratifying since we found that two single circuits when cascaded had a 2/1 slope at the asymptote. The only difference when the circuits are coupled is the starting point of the asymptote, which is now $S=1+C^2$ instead of $S=1.0$. It is apparent that two cascaded circuits produce the same curve shape as would two coupled circuits with zero coupling ($C=0$). Of course, this is rather impractical since zero coupling would mean no signal transfer. However, we may think of S_0 as having a value of 1.0 for single circuits.

Using this concept of these curves, we can predict the boundaries for any number of circuits cascaded singly, in pairs, or in combinations of these arrangements. For instance, if we cascade a single circuit and a pair of circuits, the attenuation will be

$$U=\frac{\sqrt{1+S^2}\,\sqrt{(S_{02}^2-S^2)^2+4S^2}}{S_{01}\,S_{02}^2} \tag{25}$$

where $S_{01}=1.0$, and the asymptote

$$\log U=3\log S-3\log S_{0m} \tag{26}$$

where $S_{0m}=\sqrt[3]{S_{01}\,S_{02}^2}$. Now, it should be apparent that for any number of circuits cascaded, the asymptote will be determined by

$$\log U=N\log S-N\log S_{0m} \tag{27}$$

where S_{0m} is the geometric mean of all the individual values of S_0. In other words, the boundary of any

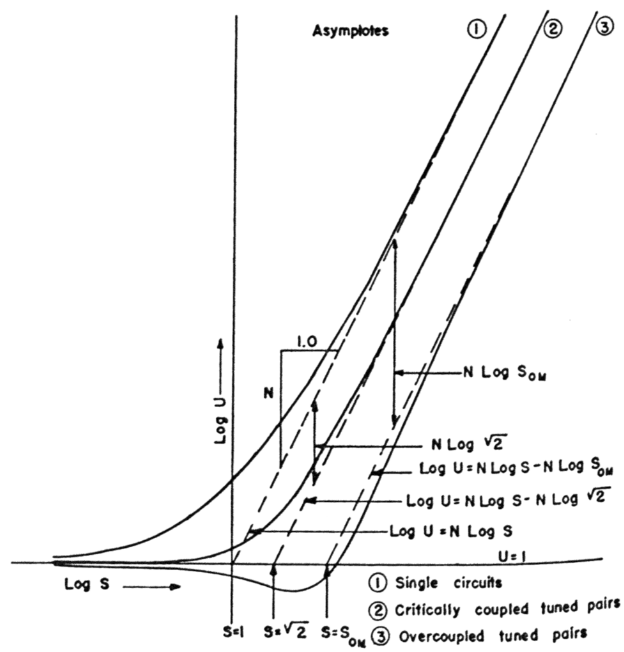

Fig. 9. Selectivity curves of cascaded circuits shown by full lines; dotted lines are asymptotes according to Equation (27)

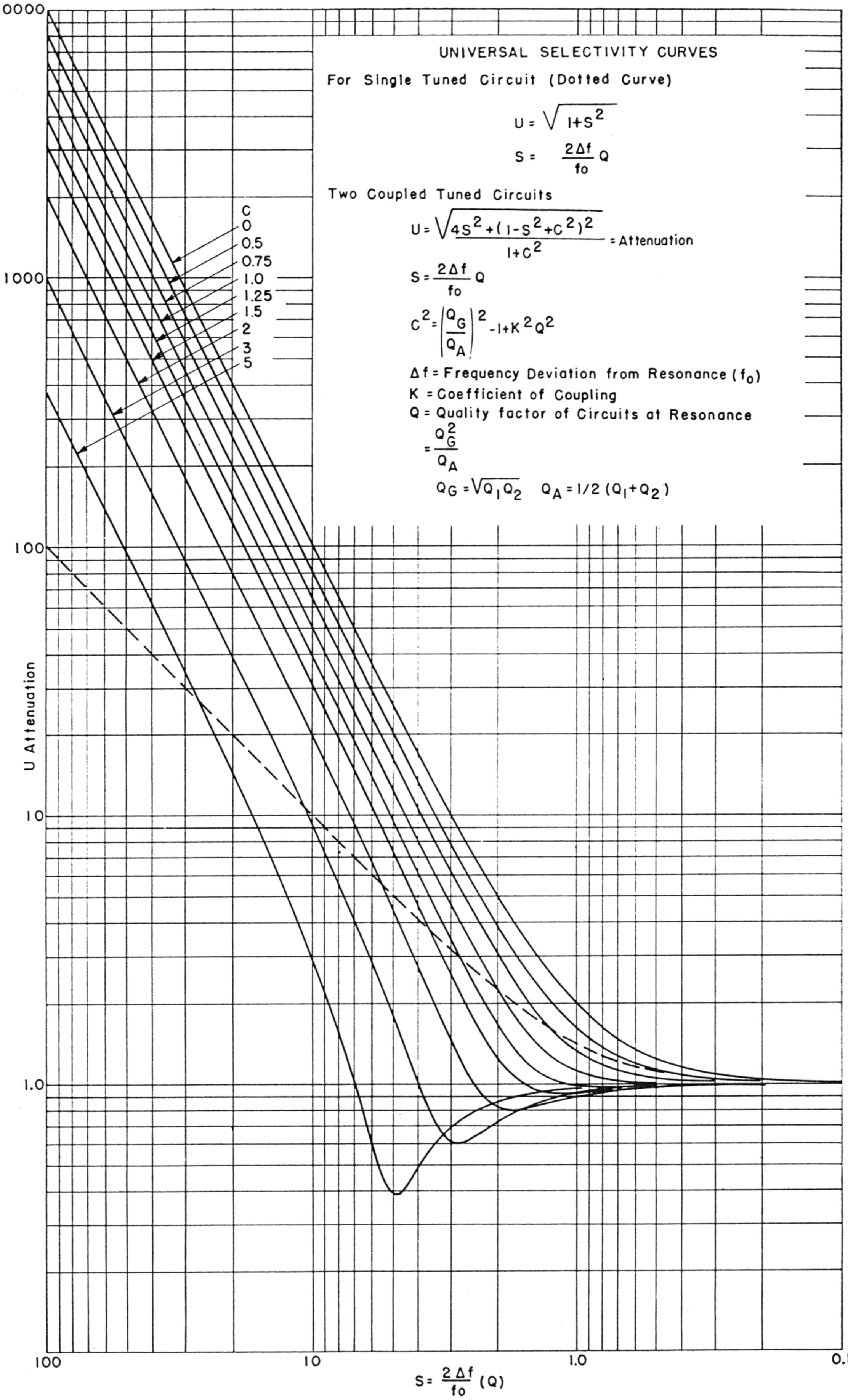

Fig. 8. Universal selectivity curves for single tuned circuit and two coupled tuned circuits

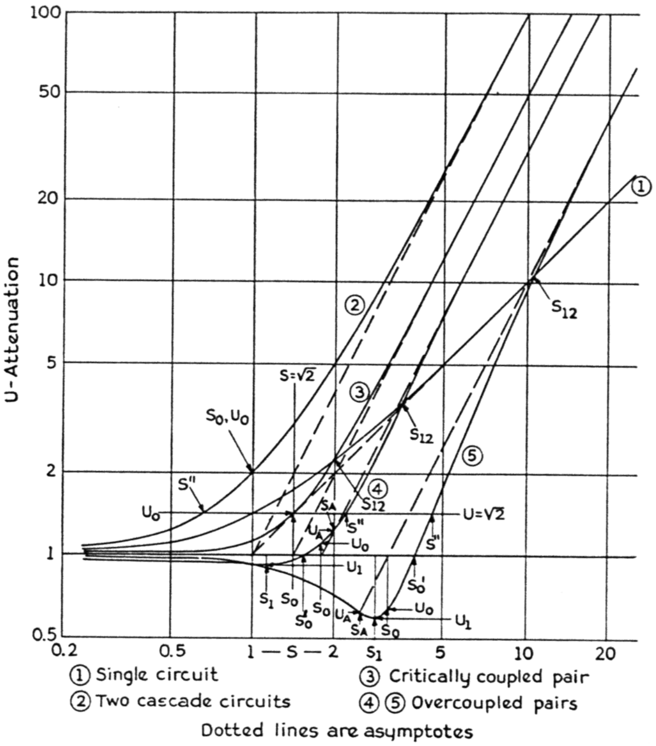

Fig. 10. Showing attenuation and deviation at various intercepts

selectivity curve produced by N single and coupled circuits will be the line $U=1$ (except for slight diversions in overcoupling) and the line starting from $S=S_{0m}$ and having the slope N when plotted on log-log paper, as shown in **Fig. 9**.

This information would allow us to set up the framework for a selectivity curve and, with a few pegs, the curve itself could be sketched in quite rapidly. For instance, we know that the curve will not be likely to pass through $S=S_0$ at $U=1$ in the case of a pair of coupled circuits, but by making this substitution in Equation (24), we find that this particular attenuation

$$U_0=2/S_0 \text{ at } S=\pm S_0 \tag{28}$$

If the circuits are overcoupled, we may determine the minimum points by setting the derivative of Equation (24) with respect to S equal to zero and solving for S. Calling this S_1 we obtain

$$S_1=\pm\sqrt{S_0^2-2}=\pm\sqrt{C^2-1} \tag{29}$$

The attenuation for $S=S_1$ from Equation (24) will be

$$U_1=2\,C/S_0^2 \tag{30}$$

In the case of overcoupled circuits, the selectivity curve crosses the $U=1$ line at a value of S we will call S_0'. Substituting $U=1$ in (24)

$$S_0'=\pm\sqrt{2}\,S_1 \tag{31}$$

Again in overcoupled circuits, the curve will cross its asymptotic line and this crossover point will occur at a value of S which may be called S_A. The equation of the asymptotic line is $U=(S/S_0)^2$. Solving this simultaneously with Equation (24) the solution for S is

$$S_A=\pm S_0^2/S_0' \tag{32}$$

TABLE I: SELECTIVITY MEASUREMENTS

xNI Attenuation	C_m=19400 mmfd Kc Deviation		C_m=9600 mmfd Kc Deviation		C_m=5200 mmfd Kc Deviation	
(U)	$-\Delta f$	$+\Delta f$	$-\Delta f$	$+\Delta f$	$-\Delta f$	$+\Delta f$
min. 0.95	—	—	2.86	2.60	—	–
min. 0.7	—	—	—	—	6.63	6.77
1	0	0	3.9	4.02	9.5	9.35
2	4.67	4.42	7.29	7.80	12.73	12.35
4	7.29	7.15	10.50	11.40	17.4	16.9
10	11.15	11.40	15.33	16.90	24.5	26.9
30	19.6	20.0	26.3	29.1	40.5	47.9
100	32.6	35.6	43.5	54.3	65.2	88.5

Substituting this value of S in $U=(S/S_0)^2$, we find the corresponding attenuation at S_A to be

$$U_A=S_A/S_0' \tag{33}$$

The intersection between the curve for a single circuit and any coupled-circuit curve may be found

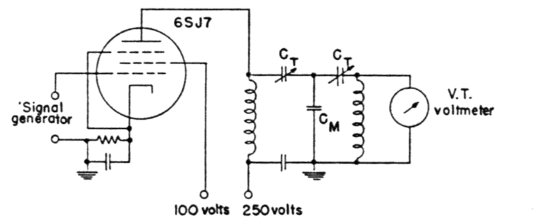

Fig. 11. This pair of circuits, using capacity coupling, were measured for selectivity and compared with the selectivity calculated for the same constants

by solving $U=\sqrt{1+S^2}$ with Equation (24) for S. The result is

$$S_{12}=\pm\sqrt{S_0^4+2\,S_1^2} \tag{34}$$

and the corresponding attenuation

$$U_{12}=\sqrt{S_0^4+2\,S_1^2+1} \tag{35}$$

A point on the curve which is often of considerable interest is that point at which the response falls to approximately 70 per cent of resonant response; in other words, where the attenuation becomes $U=\sqrt{2}$. For a single circuit, resonant frequency at this point divided by the bandwidth ($2\Delta f$) is equal to the Q of the circuit. Substituting $U=\sqrt{2}$ in Equation (24), the solution for S is

$$S''=\pm\sqrt{S_1^2+\sqrt{S_1^4+S_0^4}} \tag{36}$$

These various points are illustrated graphically in **Fig. 10**.

Checking Calculated Results

These formulas have been used several times and have been found to give results which check experiment very closely. A set-up was made including a

TABLE II: CALCULATED DATA

C_m mmfd	K %	C KQ	S_0 $\sqrt{C^2+1}$	Δf at S_0 $S_0/.37$	U_0 $2/S_0$	S_1 $\sqrt{C^2-1}$	Δf at S_1 $S_1/.37$	S'' $\sqrt{S_1^2+\sqrt{S_1^4+S_0^4}}$	Δf at S'' $S''/.37$
5200	3.04	2.81	2.98	8.05 kc	0.67	2.623	7.10 kc	4.26	11.5 kc
9600	1.65	1.52	1.82	4.92	1.10	1.148	3.10	2.21	5.97
19400	.814	.753	1.25	3.38	1.60			1.10	2.97

C_m mmfd	U_1 $2C/S_0^2$	S_0' $\sqrt{2}\ (S_1)$	Δf at S_0' $S_0'/.37$	S_{12} $\sqrt{S_0^4+2S_1^2}$	Δf at S_{12} $S_{12}/.37$	U_{12} $\sqrt{1+S_{12}}$	S_A S_0^2/S_0'	Δf at S_A $S_A/.37$	U_A S_A/S_0'
5200	.631	3.71	10.0 kc	9.63	26.0 kc	9.70	2.39	6.46 kc	0.645
9600	.981	1.625	4.38	3.69	9.98	3.82	2.04	5.51	1.252

6SJ7 amplifier, a pair of circuits coupled capacitively and a vacuum-tube voltmeter as shown in **Fig. 11**.

The inductance of the individual coils was such that they tuned to 500 kilocycles with a capacity of 168 micromicrofarads. Selectivity measurements made at 500 kilocycles are tabulated in Table I. The data are plotted in **Fig. 12**.

The measured Q of the primary coil was 87. Taking the tube plate resistance to be 1.0 megohm, the effective primary Q may be determined from the parallel impedance of the tube and the circuit at resonance

Circuit $Z = X_c Q = (1910)\ (87) = 166{,}000$ ohms

Effective primary impedance $Z_e = ZR_p/(Z+R_p) = 142{,}000$ ohms

Effective $Q = Z_e/X_c = 142{,}000/1910 = 74.5$

The measured Q of the secondary coil was 115. The tube voltmeter is assumed to have caused no effective loading on the secondary. The equivalent pair of equal Q circuits would have a Q essentially equal to the geometric mean of 115 and 74.5, which is 92.5. This value of Q may be used to relate the variable S to the frequency deviation $\Delta f(KC)$

$$S = (2\Delta f/f_0)\ Q$$
$$= 2\Delta f\ (92.5/500)$$
$$= 0.37\Delta f$$

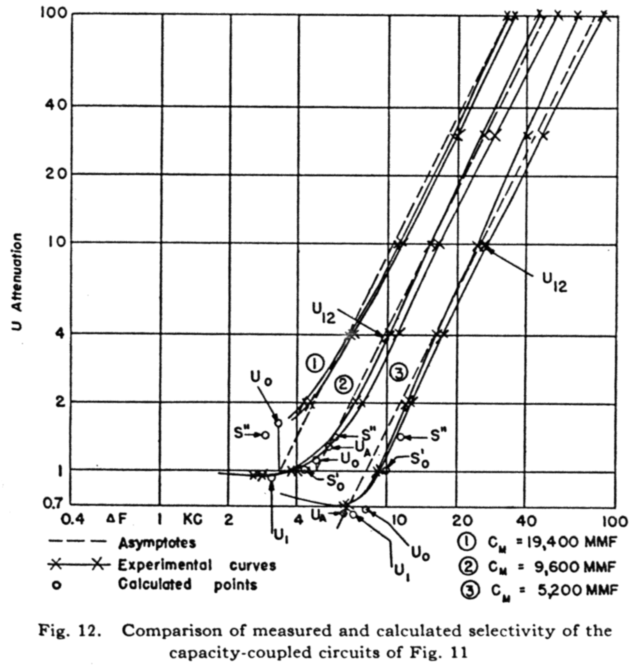

Fig. 12. Comparison of measured and calculated selectivity of the capacity-coupled circuits of Fig. 11

The coefficient of coupling is determined by the ratio between the series capacity of C_t and C_m and the coupling capacity C_m. We know, however, that it takes 168 micromicrofarads to tune each coil. Therefore, if it is assumed that 10 micromicrofarads exist in external circuits of each coil, the series capacity of C_t and C_m must always be 158 micromicrofarads and the coefficient of coupling is very nearly

$$K = 158/C_m$$

The remainder of the calculations are according to the developed formulas and are tabulated in Table II.

The data in Table I and Table II are plotted in **Fig. 12**. It will be noted that positive and negative frequency deviations from resonance are plotted together from the measured data, calculated data are symmetrical.

A short discussion of the method of selectivity measurement may be of interest. The signal generator (oscillator) in **Fig. 11** is provided with a calibrated output over a wide range of volts, millivolts, and microvolts. This signal generator is set at resonant frequency and some convenient output level on the vacuum-tube voltmeter, which can be obtained for a small signal from the signal generator, is chosen as "normal" level. The input required from the signal generator is called "normal input" (NI). The signal input is then increased to a desired attenuation level such as two times. The signal is then 2XNI or twice normal input. With this input voltage level, the signal generator is tuned first higher then lower than resonant frequency until the output falls to its "normal" level to give a $+\Delta f$ and a $-\Delta f$ reading from the frequency calibration of the signal generator. This procedure is repeated to obtain the various frequency deviations corresponding with different amounts of attenuation as in Table I.

Acknowledgment

The method of analysis and chart presentation used in this article stem, naturally, from many sources. Prominent among these sources of ideas are: editorials and papers in *Experimental Wireless and Wireless Engineer*, reports from the RCA License Laboratory, General Electric radio course 5A, and many comments and suggestions by P. C. Gardiner of Transmitter Engineering Division, General Electric Company.

(*Concluded*)